JOURNAL OF AIRCRAFT
Vol. 38, No. 6, November–December 2001

Aircraft Spacings That Produce Vortex-Free Region Below Flight Formations

Vernon J. Rossow*
NASA Ames Research Center, Moffett Field, California 94035-1000

A preliminary theoretical study is presented of a concept whereby the vortex wakes shed by a formation of aircraft can be redirected to produce a large region below the formation that is free of vortices. The redirection of the vortices is accomplished by first spacing the aircraft so that vortex pairs are formed between the wingtips of adjacent aircraft. If the aircraft spacings are within certain limits that are presented in the analysis, the self-induced velocity field of the vortex pairs causes all of the inboard vortices to move upward rather that downward. One application for such a process is the deployment of paratroopers from aircraft so that, while descending to the ground, no parachute will come near the lift-generated vortices shed by the deploying, or other nearby aircraft. The purpose of the study is to find out whether spacings between aircraft in a formation can be established so that the lift-generated vortices being trailed move upward rather than downward. Because the vortices shed by a single aircraft must always descend, attention was focused on multiple aircraft formations. It was found that the method is feasible and makes it possible to arrange close formations of aircraft so that all of the inboard trailing vortices move upward rather than downward. The two outboard vortices then travel downward at a greatly reduced velocity that depends on the number of aircraft in the formation. All of the inboard region below the formation is then free of vortices. The stability of spacings between aircraft in successful formations and the instrumentation required for implementation of the concept introduced are not treated.

Nomenclature

$A\!\!R$	=	aspect ratio of wing
b	=	wing span, ft (m)
b'	=	distance between vortex centers, ft (m)
C_L	=	lift coefficient L/qS
d	=	diameter, ft (m)
Im	=	impulse of a vortex pair, lb · s/ft (kg-s/m)
L	=	lift, lb (kg)
M_1	=	first moment of circulation, lb · s/ft (kg-s/m)
N_{ac}	=	number of aircraft in formation
q	=	$\rho U_\infty^2/2$, lb/ft^2 (kg/m^2)
r	=	radius, ft (m)
S	=	wing planform area, ft^2 (m^2)
T	=	dimensionless time, tU_∞/b_g
t	=	time, s
U_∞	=	velocity of aircraft, ft/s (m/s)
v_θ	=	circumferential swirl velocity, ft/s (m/s)
v, w	=	y and z components of velocity, ft/s (m/s)
Wt	=	weight of aircraft, lb (kg)
x	=	distance in flight direction, ft (m)
y	=	distance in spanwise direction, ft (m)
z	=	distance in vertical direction, ft (m)
Γ	=	vortex strength, ft^2/s (m^2/s)
Δy	=	horizontal across-trail distance, ft (m)
Δz	=	vertical distance, ft (m)
ρ	=	air density, slugs/ft^3 (kg/m^3)

Subscripts

cntrln	=	centerline
g	=	wake-generating aircraft
N	=	number of point vortices
outbd	=	outboard vortex
vr	=	vortex
wngtp	=	wingtip
∞	=	freesteam

Received 27 March 2001; revision received 24 July 2001; accepted for publication 25 July 2001.

*Senior Staff Scientist, Aerospace Operations Modeling Office, Mail Stop 210-10. Associate Fellow AIAA.

Introduction

AS background information on the dynamics of vortex wakes, note that a pair of vortices forms behind an aircraft in flight because the lift generated by its wings imparts downward momentum into the air surrounding the wing. Because the wingspans of aircraft are finite, the long slender body of downward moving air that trails from an aircraft wing has outboard edges that roll up into a pair of counter-rotating vortices. As indicated in Fig. 1, the vortices shed by the flaps merge or blend with other nearby vortices within a few wingspans behind the wake-generating aircraft, so that only a single pair persists into the far wake. The spanwise distance between the final two vortices is slightly less than the wingspan of the aircraft, b_g, and is usually designated by the parameter b'_g (Fig. 1). If the aircraft producing the wake is large, the flowfield of the vortex pair is also large, and the swirling velocities are intense.[1–3] As a result, the energy in these lift-generated wakes is large enough that vortex wakes pose a safety problem when aircraft need to fly near or into the wakes of preceding aircraft, as, for example, on landing and takeoff at airports.[3] Because the air surrounding a vortex pair has both upward and downward motion in different parts of its flowfield, Prandtl[4] suggested that following aircraft fly in the upwardly moving portion of vortex flowfields to conserve fuel. Since that time, numerous studies have been made of the concept as applied to the flight of birds[5] and to aircraft[6–9] for conservation of energy or fuel.

The generation, persistence, and decay of vortex wakes has also been studied[1–3,10–16] for some time to determine the nature of the hazard that vortices pose to following aircraft and to parachutes deployed from the wake-generating aircraft.[17,18] The magnitude of the hazard posed by vortices is obtained by considering the intensity of the swirling velocities in wakes. Measurements indicate that the velocity in the swirling flowfields of vortices can range from

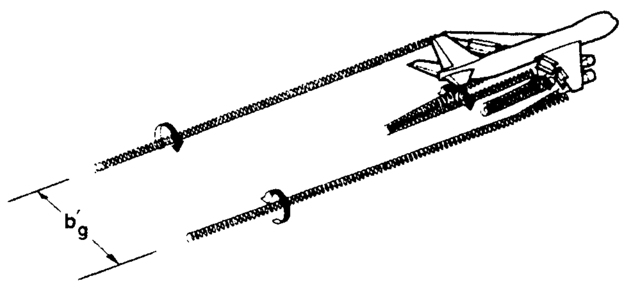

Fig. 1 Diagram of rollup of wake–vortex system shed by aircraft with landing flaps deployed.

negligible at outer radii to a sizable fraction of the flight velocity of the generating aircraft near vortex centers.[1–3] Therefore, if during the active life of vortices, another aircraft needs to pass through regions occupied by vortices, their rotary flowfields can pose a safety hazard. Another example, which relates to the subject of this paper, comes about when an aerodynamically sensitive device such as a parachute, after being deployed by an aircraft, needs to descend through a region occupied by vortices, and perhaps pass near a vortex center. The vortex flowfield may then completely dominate the parachute motion.[17,18] Because the parachute has a fall velocity in still air of about 5 ft/s, travel near a vortex flowfield with local swirl velocities of 50 ft/s or more may result in the path of the parachute being completely dominated by the vortex.

One way to prevent encounters with vortices in the flowfield under aircraft formations is to redirect as much as possible the lift-generated vortices shed by the aircraft, which usually have a tendency to descend. Because parachutists are deployed from an aircraft only when it is at least several wingspans above the ground plane, it is assumed that the lateral drift of vortices relative to the flight path is negligible. It is necessary to consider multiple aircraft formations because the vortex pair shed by a single isolated aircraft wing must always descend to represent the downward momentum imparted to the air by the lift on the wing.[3]

The objective here is to find out whether redirection of vortices can or cannot be accomplished and, if so, how to design the spacings between aircraft in a formation so that the inboard vortices go upward rather than downward after being generated. This leaves the two outboard vortices to travel downward as a representation of the total lift on all of the aircraft. If it is possible to generate such a process, a large inboard region below the aircraft formation would be free of vortices so that parachute drops from the inboard aircraft would proceed without being disrupted by the swirling flowfield of a vortex. Furthermore, because the two outboard vortices are moving downward slowly, deployment of paratroopers from the two outboard aircraft is probably also safe from vortex encounters because those vortices descend much more slowly than the parachutists. As far as the duration of the various upwardly moving vortices is concerned, previous experience with the close arrangements of aircraft spacings such as those to be studied indicates that the wakes shed by the inboard aircraft will decompose quickly.[3,19]

This paper first describes the concept to be studied and how it is analyzed. Because the motion of vortices in a closely spaced array are highly nonlinear and time dependent, the initial motion of the vortices may not always be indicative of the ultimate disposition of the vortices.[3,19] This characteristic of vortex motions makes it necessary to follow the vortices long enough that reliable conclusions can be drawn as to the final disposition of the vortices in the wake, that is, all of the inboard vortices must be moving in an upward direction that will not bring any of them near any other vortices that could redirect them. Therefore, the results presented consist of numerical examples that illustrate how the motion of vortex systems are affected by a range of lateral and vertical spacings between the centerlines of adjacent aircraft. These numerical examples are then used to determine which spacings provide a vortex-free region below aircraft formations and which do not. Because aircraft in successful formations will be flying in close proximity to each other, the flowfields surrounding the wings of adjacent aircraft may interact so that the span loadings and rollup of the wake vorticity might differ from that experienced by an aircraft in isolation. Computations are presented to indicate whether the rollup and subsequent motion of wing wakes proceed somewhat as they do in isolation, or if significant adjustments in the aircraft lateral/vertical spacings need to be made when the recommendations found here are applied to flight situations. Application of the wake redirection concept to parachute deployment and to wake–vortex avoidance at airports are then briefly discussed.

To have a manageable problem to analyze, it is assumed in this preliminary study that the air is inviscid and incompressible and that the vortices can be considered as having core diameters much less than the wingspan of the aircraft. In this way, the vortex systems can be treated as if they are composed of two-dimensional point vortices. It is also assumed that the wind velocity components and atmospheric nonuniformities are negligible. In the computations to be presented, the various parameters are made dimensionless by use of the wingspan b_g, the flight velocity U_∞ of the aircraft, and the time t. One time unit, $T = tU_\infty/b_g$, is then equal to one wingspan of travel by the aircraft along its flight path. In particular, distances along the axes of the coordinate system, which are fixed relative to the ground at the same altitude as the leading aircraft, become x/b_g, y/b_g, and z/b_g. Similarly, the velocity components are combined with the flight velocity of the aircraft, as u/U_∞, v/U_∞, and w/U_∞. The total circulation in each vortex is made dimensionless by combining it with the wingspan and flight velocity of the aircraft as $\Gamma/b_g U_\infty$. As shown later, the dimensionless parameters are usually written out to aid in the interpretation of the results.

Concept Being Studied

Consider a vortex array shed by a formation of four aircraft as they fly side by side at the same altitude (Fig. 2). The rotary flowfield associated with each vortex in the array induces an upward or downward velocity on each of the other vortices, as indicated by the circular arrow for each vortex. Because the vortices in the array shown in Fig. 2 are all equally spaced and of the same strength, the net velocity induced on the vortices near the center of the array is nearly zero because the upward and downward contributions are about equal. Near the ends of the vortex array, the downward contributions dominate.

It is reasoned that some reduced and probably small spanwise spacing between aircraft centerlines (and, therefore, between wingtips and, consequently, vortices) will produce a configuration where the initial velocities of all of the inboard vortices will be upward and persist in an upward motion without later changing to a downward direction. The analysis to be presented determines the spanwise and vertical spacings between adjacent aircraft in a formation that will produce upward motion of the inboard vortices. To determine the range of acceptable lateral and vertical spacings between adjacent aircraft, numerical simulations are carried out on several sequences of flight formations. The different wake motions produced by the sequential spacings are then used to indicate acceptable spanwise, vertical, and in-trail spacings. In the analysis to be described, it is assumed that all of the aircraft in the formation are the same, that their slat and flap configurations are about the same, and that spacings between adjacent aircraft are all the same.

Analysis Tools

The swirl velocity in the flowfield around a single isolated vortex is defined by a circulation parameter Γ. The circumferential velocity

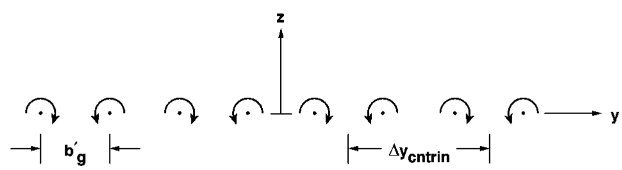

Fig. 2 View in direction of flight of array of vortex pairs shed by four aircraft flying side by side.

v_θ in the vortex is then related to the circulation and to the radius r from the center of the axially symmetric swirling flowfield by

$$v_\theta = \Gamma/2\pi r \tag{1}$$

Equation (1) applies as long as the radius of the vortex core is less than the distance from the vortex center to the location where the velocity is being applied. The vortices that trail from a lifting wing (Fig. 1) come about because the finite span of the wing allows the air at higher pressure below the wing to flow around the wingtip edges into the lower pressures on top of the wing to form a pair of counter-rotating vortices. In the material presented here, the vortex shed by the starboard wing is taken as positive and that for the port wing as negative, where the magnitude of the circulation in each is the same. The circulation Γ in a vortex is determined by the amount of lift on the generating wing by the Kutta–Joukowsky theorem as

$$L = Wt = \rho U_\infty b'_g \Gamma \tag{2a}$$

where b'_g is the spanwise distance between the centers of the rolled up vortices. For a number of subsonic transports in their low-speed landing configuration with flaps deployed, the dimensionless value of the circulation is approximately

$$\Gamma/b_g U_\infty \approx 0.1 \tag{2b}$$

which is used as typical in the computations presented later. Whenever only one vortex pair is in the flowfield, each vortex induces the same magnitude of downward velocity on the other vortex in the pair. The motion is then in a direction perpendicular to the line drawn between the centers of the vortices. Furthermore, the magnitude and direction of the motion of the downward moving pair is constant until the flowfield begins to decompose and is given by a combination of Eqs. (1) and (2) as

$$w_{\text{vr}} = \frac{-\Gamma}{2\pi b'_g} = \frac{-Wt}{2\pi\rho U_\infty b'^2_g} \tag{3}$$

where w_{vr} is the descent velocity of the vortex pair. A negative sign is used to indicate that the vortices are moving downward or in the negative z direction. The spanwise distance between the rolled up vortices for an elliptically loaded wing is given by $b'_g = b_g\pi/4$, which is a good approximation to many other conventional span loadings and will be used as representative in the examples to follow.[20]

The first moment of the circulation about the centerline of the flowfield in the point-vortex representation being studied is given by

$$M_1 = \rho \sum_{j=1}^{N} y_i \Gamma_i \tag{4}$$

which is the same as the impulse of the vortex system.[20] When Eq. (4) is restricted to a single vortex pair centered at a lateral distance y_c from the centerline of the vortex system, its first moment is given by

$$M_1 = \rho[(y_c + b'_g/2)\Gamma - (y_c - b'_g/2)\Gamma] = \rho\Gamma b'_g \tag{5a}$$

which is the same as the impulse for an isolated vortex pair, or

$$M_1 = Im_1 = \rho b'_g \Gamma \tag{5b}$$

The impulse given by Eqs. (4) and (5) will be used later to evaluate the downward velocity of the two outboard vortices that are shed by the aircraft formations being studied.

It is found that the descent velocity of a single vortex pair in a uniform stationary atmosphere for subsonic transports ranges from several feet per second for small turboprop commuter aircraft to around 10 ft/s for the largest and heaviest transport aircraft now in the air-transportation fleet. Observations of aircraft wakes in the atmosphere also indicate that the dispersion of the vortex flowfield and atmospheric structure appear to limit the descent distance of the vortices to about 1000–1500 ft when observed at cruise altitudes. Near the ground, the descent distance is usually less because the ground surface diverts the downward motion of the vortices to a sideways motion. It has also been observed that the vortices may still retain substantial swirl regions even though they no longer descend or move.

Because the strength of the vortex or its circulation Γ depends on the spanwise distribution of and total lift on the wake-generating wing, the dimensionless circulation in the vortex is estimated as

$$\Gamma/b_g U_\infty = \tfrac{1}{2}(b_g/b'_g)(C_{Lg}/AR_g) \tag{6}$$

where C_{Lg} is the lift coefficient and AR_g the aspect ratio of the wing. As pointed out for Eq. (2b), the dimensionless circulation for a subsonic transport in its landing configuration is usually around 0.1. The downward velocity of a vortex pair may then be written as

$$w_{\text{vr}}/U_\infty = -(1/4\pi)(b_g/b'_g)^2(C_{Lg}/AR_g) \approx -(0.2/\pi^2) \approx 0.02 \tag{7}$$

The analysis described here deals with a number of vortex wakes or pairs that interact strongly with each other. Because the vortex flowfields are embedded in the air, they move with the local velocity of the air. The motion of the vortices appears as an interaction between the flowfields of the individual vortices and the ambient wind, which is assumed here to be negligible. These interactions are analyzed in a Trefftz-plane approximation, where the vortices are treated as two dimensional, and the along-vortex variations are assumed negligible. The velocity field of the vortices then consists of components in the spanwise and vertical velocity directions as induced on the ith vortex by all of the other vortices in the flowfield:

$$v_i = -\frac{1}{2\pi}\sum_{j=1}^{N}\frac{\Gamma_i(z_i - z_j)}{(y_i - y_j)^2 + (z_i - z_j)^2} \tag{8a}$$

$$w_i = +\frac{1}{2\pi}\sum_{j=1}^{N}\frac{\Gamma_i(y_i - y_j)}{(y_i - y_j)^2 + (z_i - z_j)^2} \tag{8b}$$

The strengths of the point vortices are given by $\Gamma_i/b_g U_\infty = 0.1$, as indicated in Eq. (2b), and locations by (y_i, z_i). In the computations, all of the various parameters are made dimensionless as described earlier. If the locations where the computations are being made are near the end of a vortex segment, or a more precise analysis is desired, the effect of the finite length of vortex segments is accounted for by use of equations presented by Glauert.[21]

Illustration of Concept

To demonstrate the concept, the computed motion of the vortices shed by a single aircraft is first presented in Fig. 3a. Note that the vortex motions are displayed here as a view from behind the generating aircraft, rather than as a slantwise view of the vortex lines, which is difficult to interpret. The vortex paths shown were generated by numerical integration along the paths of the vortices as time proceeds. As expected, the motion shown in Fig. 3a is a downward one, which is constant in velocity to form straight paths, because the vortex pair is isolated from other vortex pairs and from the ground. Note that for aircraft in their landing configuration where $\Gamma/b_g U_\infty \approx 0.1$, and as predicted by Eq. (7), the self-induced velocity field of the vortex pair shed by a single aircraft typically causes the vortices to move downward about two wingspans during the time t that the aircraft travels along its flight path for 100 spans, that is, $T = tU_\infty/b_g = 100$ (Fig. 3a).

Consider next the case where four aircraft are flying abreast at two wingspans between adjacent centerlines. As shown in Fig. 3b, the vortex paths are no longer necessarily straight when multiple vortex pairs, that is, multiple aircraft, are in the flowfield. Also note that all of the vortices have a generally downward velocity that differs slightly. An upward motion of the inboard vortices is not achieved because the spanwise spacing between adjacent aircraft is too large.

Based on information presented in sections to follow, the wake–vortex motions computed for a successful solution are presented in Fig. 3c for six aircraft flying abreast. In this case, the distance between the centerlines of adjacent aircraft in the formation has been decreased from the $2b_g$ used in Fig. 3b to $1.2b_g$, that is, $0.2b_g$ between wingtips. The vertical spacing between the centerlines of

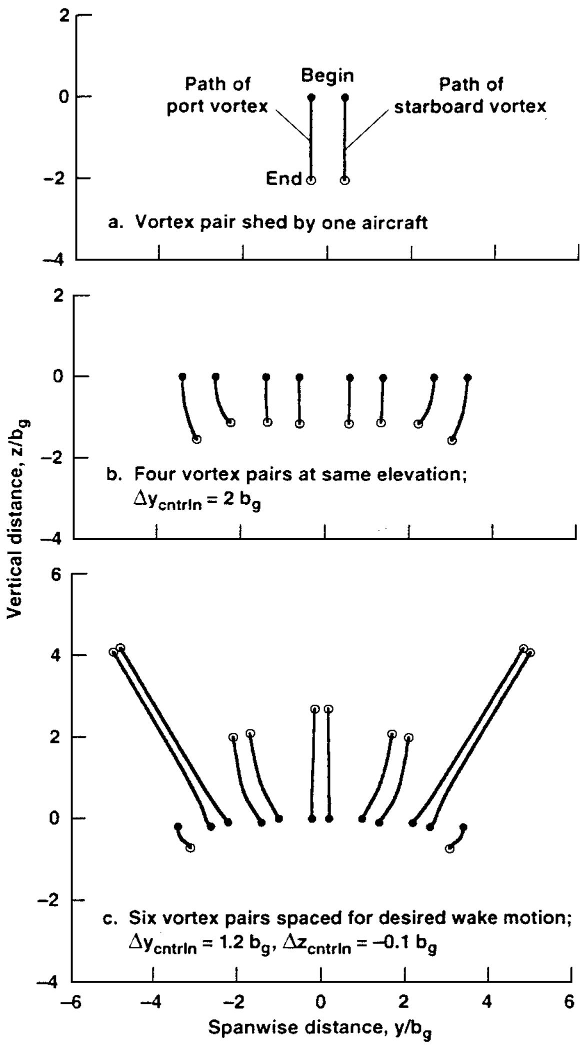

Fig. 3 View in direction of flight of computed vortex paths as they move from initial location to the one predicted for when aircraft has moved 100 spans in flight direction.

adjacent aircraft wherein a more outboard aircraft is below its adjacent inboard aircraft by $0.1b_g$, that is, $\Delta z_{\mathrm{cntrln}} = -0.1b_g$. As a consequence, all of the inboard vortices shed by the formation move in an upward direction that spreads the vortex pairs laterally so that they will not likely change direction, and none move downward (Fig. 3c). In the same configuration, the two outboard vortices have a markedly reduced downward velocity (compare with Fig. 3a). In other words, the flight formation successfully manages its wake vortices to achieve a vortex-free region below the formation and in between the two outboard vortices, which are nearly stationary. The vortex dynamics found in Fig. 3c is the one desired for parachute deployment and perhaps for other wake–vortex problems as well.

A determination of the ranges in acceptable spacings between adjacent aircraft in flight formations that will guarantee the same kind of wake motion shown in Fig. 3c is the goal of the research presented here. To achieve this goal, the dynamics of multiple, vortex–wake systems are presented for a sequence of aircraft formations to identify the boundaries of acceptable aircraft spacings. Because the separations between aircraft needed to achieve the desired wake–vortex dynamics are much less than one wingspan, it will probably be necessary to utilize position-keeping electronic guidance on all of the aircraft in the formation to maintain the needed position accuracy.

Velocity of Two Outboard Vortices

The downward momentum imparted to the air by the lift on aircraft wings is represented by the downward impulse in each vortex pair. The total impulse or momentum of the downward moving air in the system must remain constant with time, even though individual vortices may move all over the flowfield. The conservation of downward momentum in the air, as represented by the first moment of circulation, or impulse of vortex pairs[20] [Eqs. (4) and (5)], is now used to determine the eventual lateral location and downward velocity of the two outboard vortices. As before, it is assumed that the vortices are concentrated so that they can be approximated by point vortices. The first moment or downward impulse for the entire formation is the sum of the separate impulses for all of the aircraft in the formation given by either Eqs. (4) or (5b) through summation as

$$Im_{N_{\mathrm{ac}}} = \rho b_g' \Gamma N_{\mathrm{ac}} \tag{9}$$

where N_{ac} is the number of aircraft in the formation.

If the design of the formation is successful in causing all of the inboard vortices to move upward by an appreciable distance, the upward moving pairs become more remote from the two outboard vortices, which also have a decreasing distance between them. As a consequence, the two outboard vortices act more and more as if they are isolated from the upward moving vortex pairs. As time progresses, the entire first moment of circulation or downward impulse given to the air by the wings of the aircraft is more nearly represented by the spanwise distance between the two outboard vortices, which are moving downward slowly. Because the downward impulse of the initial and final spanwise spacings must be the same, the spanwise spacings between the two outboard vortices is given by Eqs. (4) or (5b) as

$$b_g'|_{\mathrm{outbd}} \approx b_g' N_{\mathrm{ac}} \tag{10}$$

and the final downward velocity of the outboard pair becomes

$$w_{\mathrm{outbd}} \approx \Gamma / 2\pi b_g' N_{\mathrm{ac}} \tag{11}$$

That is, the downward velocity of the outboard two vortices is reduced by $1/N_{\mathrm{ac}}$ times the downward velocity of a vortex pair shed by a single aircraft in isolation. As a consequence, larger aircraft formations produce outboard vortices that descend very slowly. Note in Fig. 3c that the outboard vortices are still in the process of moving inboard to their final spanwise location, but have already slowed considerably in their descent. This same result is found in all of the successful formations studied.

Design of Vortex Systems

As mentioned in the foregoing sections, the goal of the study is to develop guidelines for the design of aircraft formations so that the inboard vortices shed by the group move upward, leaving a region below the formation and in between the two outboard vortices free of vortices. The purpose of this section is to present several sequences of numerical examples that lead to the guidelines needed for the spacings between aircraft in close formation. Each figure in the sequence is found by numerical computation wherein a single design parameter is changed systematically. This process makes it possible to define approximate boundaries for when the various spacing parameters produce the vortex motions that yield a vortex-free airspace below the aircraft formation.

Because the equations used in the analysis are simple in form, it may be reasoned that a direct solution based on the initial motion of the vortices is the way to solve the problem. If successful, it would not be necessary to carry out a numerical analysis that follows the motion of the various vortices for every configuration. It turns out, however, that the motion of a mixture of positive and negative vortices is so nonlinear that vortices often begin in an upward direction and then turn and go downward rapidly. Such motions of vortex pairs are unacceptable because they would pose a hazard beneath the aircraft formation, which is to be kept vortex free. Examples of these kinds of vortex motions are presented in Figs. 11–14 of Ref. 19. Those figures illustrate how vortex arrays designed for one of two types of motion can instead result in vortex trajectories quite different from the one designed on the basis of conditions at the beginning of the motion. It is concluded, therefore, that motion of all of the vortices in the system must be tracked by numerical computation long enough that the ultimate disposition of the vortices is clearly indicated.

Effect of Spanwise Spacing on Vortex Motion

As mentioned earlier in connection with Eq. (3), the spacing between the two vortices in the pair shed by an isolated wing is less than the wingspan by $b_g' \approx b_g \pi/4$. Therefore, if the vortex of an adjacent aircraft is to be more influential than the opposite vortex in its pair, the maximum spacing between the wingtip vortices of adjacent aircraft must be less than b_g'. If the initial motion of the inboard vortices is to be upward, the distance between aircraft centerlines,

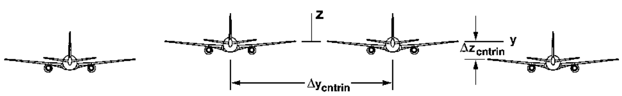

Fig. 4 View in direction of flight of four aircraft to illustrate nomenclature used for lateral and vertical spacings between aircraft centerlines.

Δy_{cntrln}, must then be less than

$$\Delta y_{\text{cntrln}} < 2b_g' \tag{12}$$

which requires that the spanwise distance between the wingtips of adjacent aircraft be less than

$$\Delta y_{\text{wngtp}} < (2b_g' - b_g) = b_g(\pi/2 - 1) \approx 0.57b_g \tag{13}$$

which is well within the capabilities of electronics to maintain a given station relative to other aircraft. At a spacing between wingtips of $0.57b_g$ ($\Delta y_{\text{cntrln}} = 1.57b_g$), the upward and downward induced velocities on a vortex are roughly equal so that the inboard vortices remain about stationary. If the centerline spacing between adjacent aircraft increases above $1.57b_g$, the vortex system descends as indicated in Fig. 3b. If the centerline spacing decreases below $1.57b_g$, the velocity field of the vortex shed by an adjacent aircraft dominates the motion so that they ascend and have the potential to produce favorable results similar to those shown in Fig 3c.

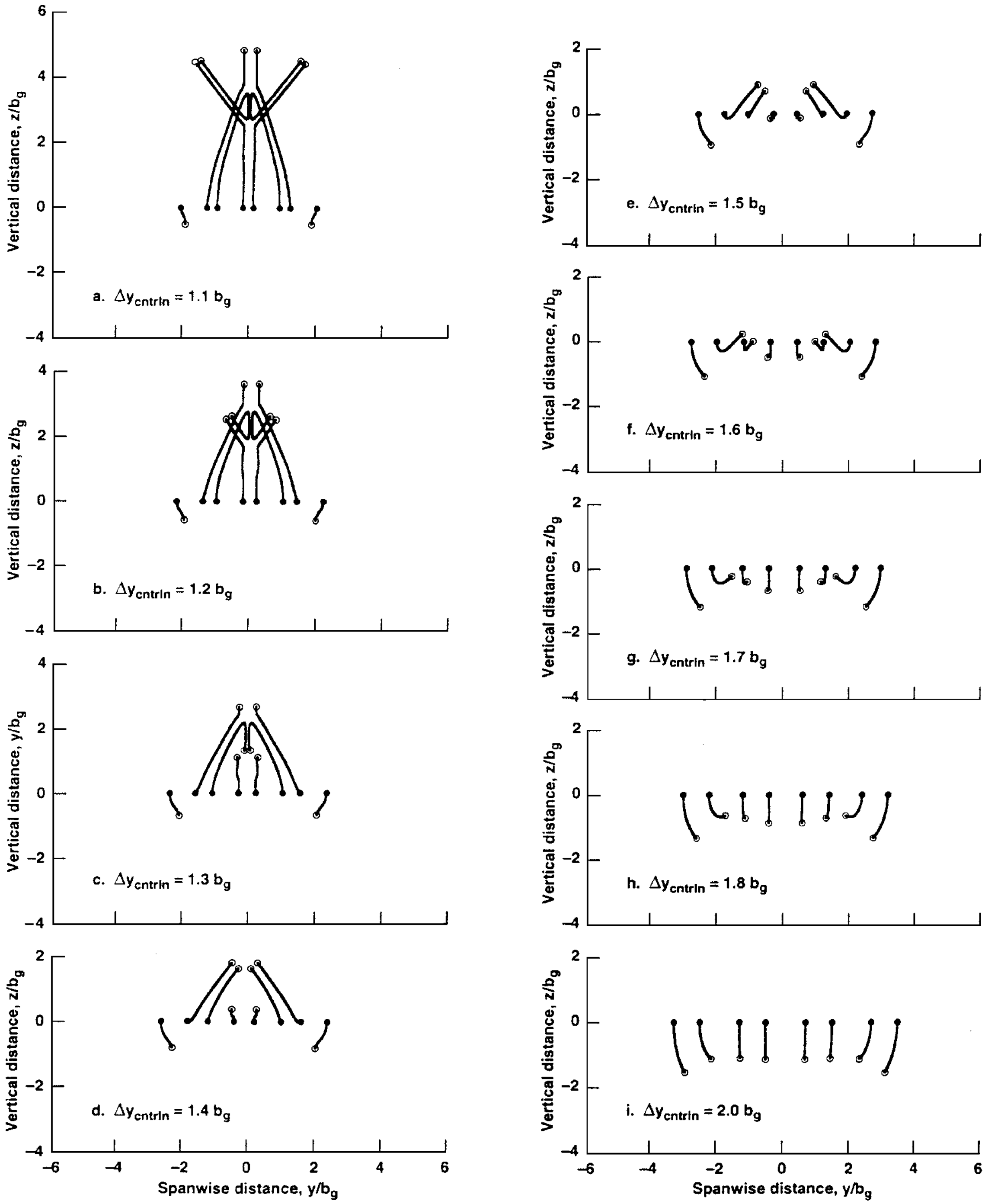

Fig. 5 Wake–vortex motions for sequence of spanwise spacings between adjacent aircraft to determine acceptable values, $\Delta z_{\text{cntrln}} = 0$.

The first sequence of configurations is a simulation of the effect of lateral spacing between adjacent aircraft on wake vortex dynamics. To isolate the lateral spacing effect on vortex motion, all of the wake-generating aircraft are placed at the same elevation ($\Delta z_{\mathrm{cntrln}} = 0$) and at the same centerline spacing between adjacent aircraft, $\Delta y_{\mathrm{cntrln}}$ (Fig. 4). The sequence of computations presented in Fig. 5 shows how the dynamics of the vortices change as the wingtip (or centerline) spacings are increased from $\Delta y_{\mathrm{cntrln}} = 1.1b_g$ to $2b_g$. The initial upwardly directed vortex motions shown in Fig. 5a–5d indicate that it is not significantly beneficial to decrease wingtip spacings much below about $0.2b_g$, which is equivalent to aircraft centerline spacings of $\Delta y_{\mathrm{cntrln}} = 1.2b_g$. When the lateral or spanwise separation between aircraft centerlines exceeds about $\Delta y_{\mathrm{ctrln}} = 1.5b_g$, the entire system of vortices moves in a general downward direction, indicating that the spacing must be less than about $\Delta y_{\mathrm{cntrln}} = 1.4b_g$, if a general upward motion of inboard vortices is to be realized. A spacing of much less than $\Delta y_{\mathrm{cntrln}} = 1.1b_g$ appears to be unnecessary to achieve an energetic upward motion. Because a spacing of $\Delta y_{\mathrm{cntrln}} = 1.2b_g$ appears to be a value roughly centered on those that are acceptable, it was chosen as the basis for the study of the effect of other parameters on wake dynamics.

Effect of Vertical Spacing on Vortex Motion

The results presented in Fig. 5 are a mixture of success and failure. The configurations being sought are those where all of the inboard vortices initially move upward and then continue to do so. In Figs. 5a–5c, however, the vortices first move upward, then inboard, and finally some have a rapid downward movement that could pose a severe hazard to users of the airspace below the flight formation. The inboard component of the upwardly directed vortices appears to bring vortices into the proximity of each other so that a downward motion is given to some of the pairs. The cases shown in Figs. 5a–5c exhibit such motion, and Fig. 5d appears about to generate the same kind of motion at a time greater than $T = 100$. Although all of the inboard vortices have an initial upward velocity, the subsequent motion suggests that some of the vortices will later turn downward, which could then pose a safety problem.

Examination of the trajectories of the vortices in these cases, suggests that the initial velocity of the vortices should be given a slight outboard component to counteract the inward motion shown in Figs. 5a–5e that can lead to the reversal in direction of the vortex motion (Figs. 5a–5c). In this subsection, cases are presented to illustrate how vertical displacement between the centerlines of adjacent aircraft can be used to eliminate the reversal in vertical direction that sometimes occurs when all of the aircraft, and their shed vortices, are at the same elevation.

The logic behind the use of vertical displacement distances between adjacent aircraft is shown in Fig. 6. Note that even at modest vertical displacements, the initial velocity direction of a vortex pair can be given a significant velocity component in the spanwise direction. In fact, the wingtip vortices of adjacent aircraft are so close together at the outset that, even small changes in vertical height cause large changes in the resulting vortex dynamics. Therefore, the range of acceptable vertical separations is not large, and small increments between cases are needed to clearly define the guideline boundaries.

The numerical examples presented in Fig. 7 show how sensitive the vortex paths are to vertical displacements between aircraft centerlines. The results first demonstrate that acceptable flight-formation spacings are not achieved with positive, or zero, vertical displacements between adjacent aircraft (Figs. 7a–7d). It is not until a negative vertical displacement of $\Delta z_{\mathrm{cntrln}} = -0.025b_g$ (Fig. 7e) is used that the inboard region below the aircraft formation can be guaranteed permanently vortex free. It is concluded that an outboard tilt of the velocity vectors (and the vortex paths) appears necessary to accomplish a permanent upward direction to the vortex motions. The maximum vertical displacement that appears to be allowable (when $\Delta y_{\mathrm{cntrln}} = 1.2b_g$) is around $\Delta z_{\mathrm{cntrln}} = -0.15b_g$ (Fig. 7h). An illustration of a concept carried too far is presented in Fig. 7j, where the outboard vortices swing downward and inboard to contaminate the inboard region below the flight formation that was to be protected. The recommended range of vertical displacements is then estimated as $\Delta z_{\mathrm{cntrln}} = -0.025b_g$ to $-0.15b_g$. A midrange value of $\Delta z_{\mathrm{cntrln}} = -0.10b_g$ is recommended as a target value, so that small variations in aircraft positions still retain the desired wake motions.

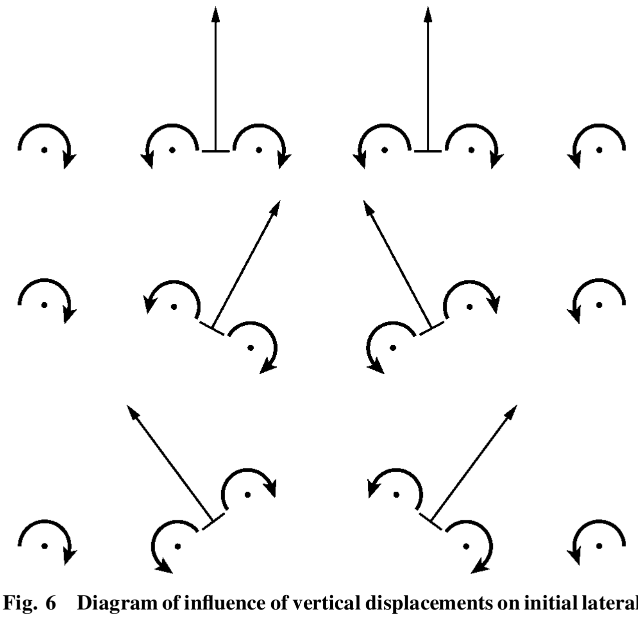

Fig. 6 Diagram of influence of vertical displacements on initial lateral motion of vortices.

Number of Aircraft in Formation

The two foregoing subsections indicate that spacings between the centerlines of adjacent aircraft should be kept between about $1.1b_g$ and $1.4b_g$ in the horizontal and between $-0.025b_g$ and $-0.15b_g$ in the vertical spacings between adjacent aircraft if the airspace below the aircraft formation is to be kept vortex free. In Fig. 8, the foregoing spacings are tested at midrange values for a sequence of different numbers of aircraft in the formation. As recommended in the preceding subsection, values of the horizontal and vertical separations between the centerlines of adjacent aircraft were all set at $1.2b_g$ and $-0.1b_g$, respectively. The wake dynamics predicted by numerical computation for 2–8 aircraft in a formation are presented in Fig. 8. Note that all of the cases studied have the desired upwardly spreading wake motions for the inboard vortices that leave a vortex-free region below the flight formation. Also, in all cases, the descent of the two outboard vortices is substantially slowed.

Fore and Aft Spacing

Observations of flight formations of multiple aircraft indicate that few consist of aircraft flying side by side. The more common arrangement appears to be one where aircraft are placed fore and aft of each other chevron style, as shown in the plan view in Fig. 9. For those cases where fore and aft displacements between adjacent aircraft are desired, this subsection describes briefly how they affect the recommended lateral and vertical separations found in preceding subsections. Because the presence of aircraft wings has a significant influence on the motion of vortices near the aircraft, a spanwise vortex for each aircraft is included in the numerical computations.[21] The spanwise segment of a line vortex simulates the lift on the wings and, thereby, includes the effect of strong downwash just behind wings on vortex motions while the vortex segments are still in the near field of the aircraft. In this way, an improved estimate is obtained for the vertical locations of the vortex lines as they pass beside the more rearward aircraft in the formation. Because vortex motions are of interest, any changes in the vortex-induced lift or rolling-moment changes with stagger are ignored in this computation. Note that the outer part of the velocity field of the vortex increases in magnitude as the stagger distance increases. Also the magnitude and characteristics of those velocity changes can be computed by use of equations

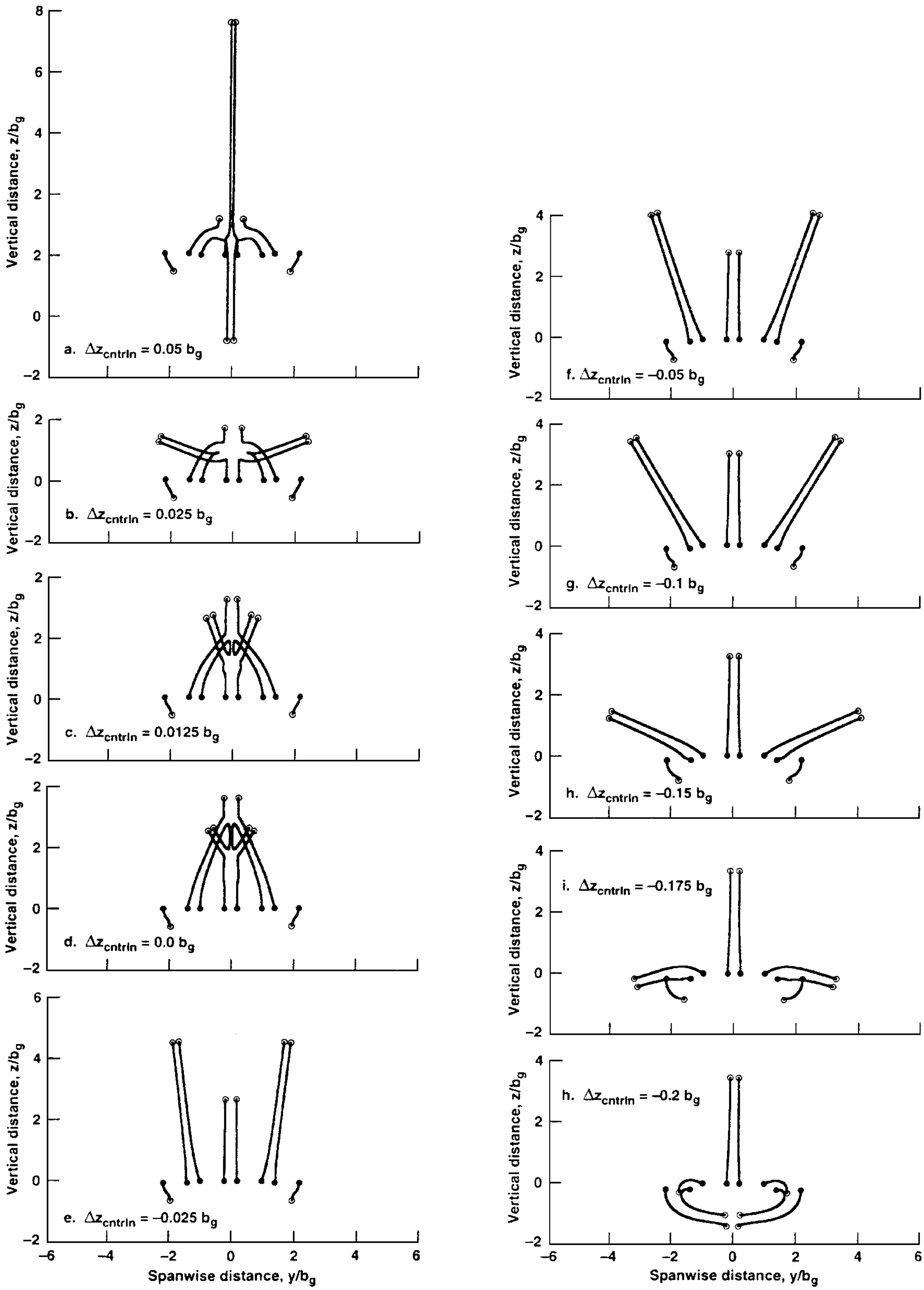

Fig. 7 Wake-vortex motions for sequence of vertical spacings between adjacent aircraft to determine acceptable values, $\Delta y_{\mathrm{cntrln}} = 1.2 b_g$.

for vortex segments that represent the span loading on the wing and the flowfield of segmented trailing vortices.[21]

When computations were carried out on various fore and aft displacements of adjacent aircraft, it was found that the dynamics of the vortices were not greatly altered (results not shown because they duplicate those already presented in Fig. 8). That is, if the fore and aft displacements between adjacent aircraft are less than about four wingspans, neither the lateral nor the vertical separation distances recommended in the preceding subsections need to be modified. If, however, fore and aft displacements larger than 4 wingspans, for example, 10 wingspans, are used, it becomes necessary to adjust the aircraft spacings by the lateral and vertical distances traveled by vortices during that time. When adjustments of that magnitude are made, the recommended spacings between adjacent vortices are restored for the times during which the vortices interact strongly. The computed vortex paths are then almost identical to those computed for side-by-side aircraft formations. In brief, the same vortex dynamics illustrated in Fig. 8 for side-by-side formations can also be

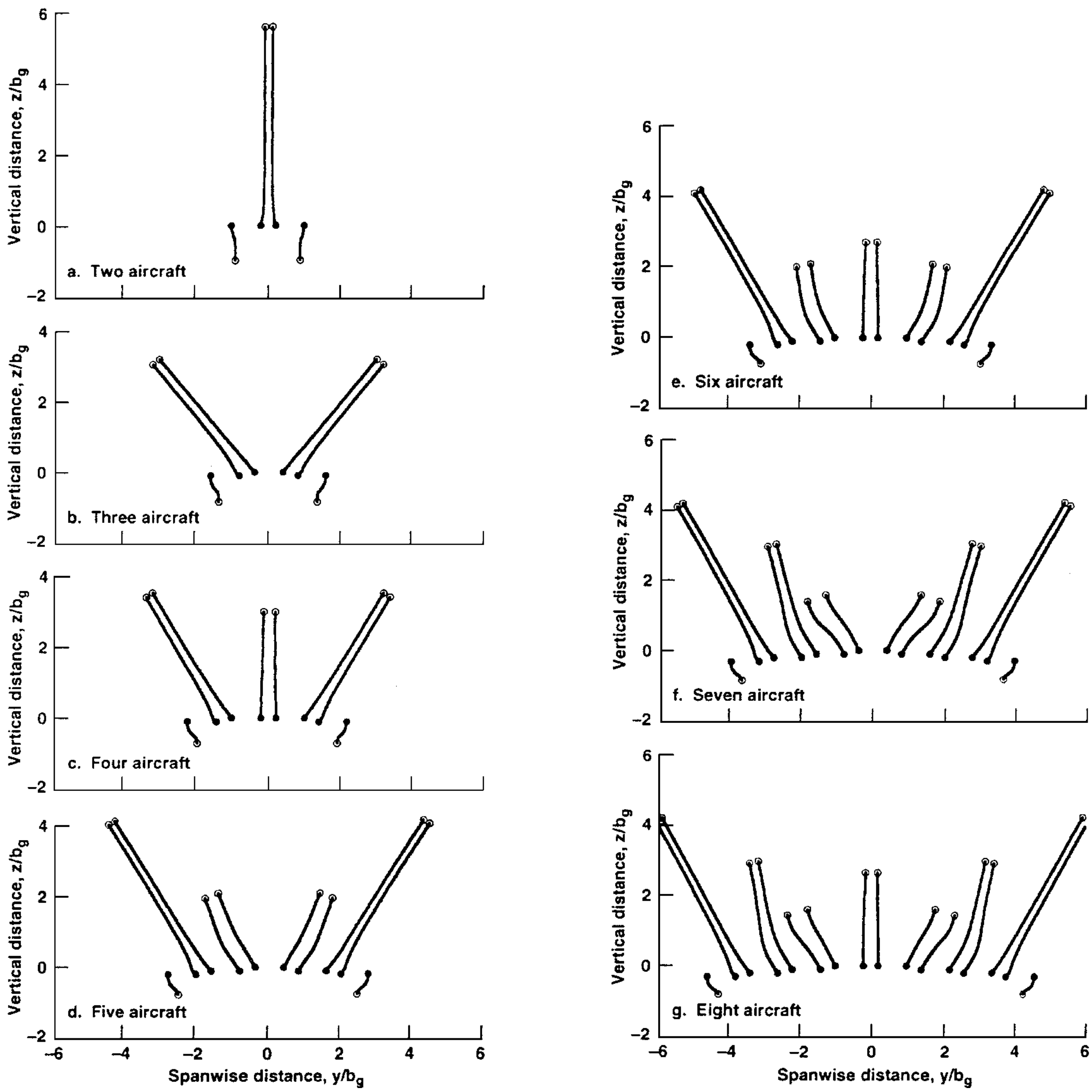

Fig. 8 Wake–vortex motions for various numbers of aircraft in the formation, $\Delta y_{\mathrm{cntrln}} = 1.2b_g$ and $\Delta z_{\mathrm{cntrln}} = -0.1b_g$.

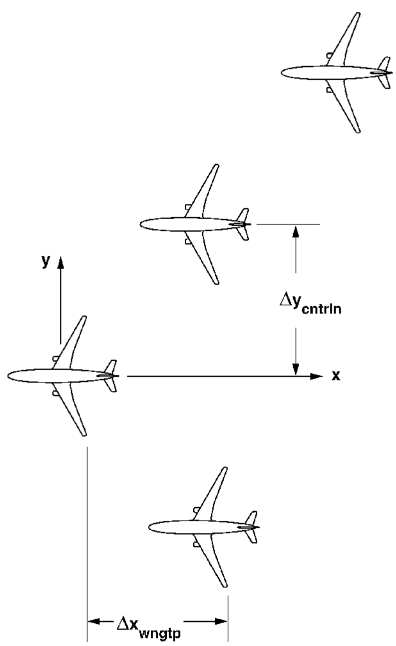

Fig. 9 Plan view of aircraft formation illustrating nomenclature used for spanwise and flight-direction displacements between adjacent aircraft.

achieved when fore and aft separations are also used, if vortex motions before passage of following aircraft are adequately included in the placement of following adjacent aircraft. The ability to use fore and aft displacements of aircraft in a formation makes the concept less hazardous to fly, so that any inadvertent variations in lateral spacings between aircraft are not so likely to lead to contact between aircraft. If vortex lift is included in the decision to place a formation into a chevron or staggered formation, there is a small increase in the vortex-induced lift benefit received by following aircraft when they are in a more aft location than directly abeam of their neighbors. Because the vortex flowfield becomes close to two dimensional a few spans downstream of the generating wing, maximum benefits are probably realized at spacings in the flight direction of about four spans.

Wake Rollup Interference Between Adjacent Aircraft

When lifting surfaces are near one another, the flowfield of one wing has an influence on the lift distribution (and structure of shed vortices) of the other aircraft wing. To explore the changes expected in a rolled-up wake due to the proximity of nearby wings, two situations are compared in Figs. 10 and 11. In Fig. 10, the rollup of an elliptically loaded wing that is isolated from all other wings is presented. In this example, each subvortex in the span loadings has the same strength. Note that the rolled-up vortex for a wing in isolation results in an accumulation of most of the point vortices in one small locality, which results in small tight vortex cores that descend.

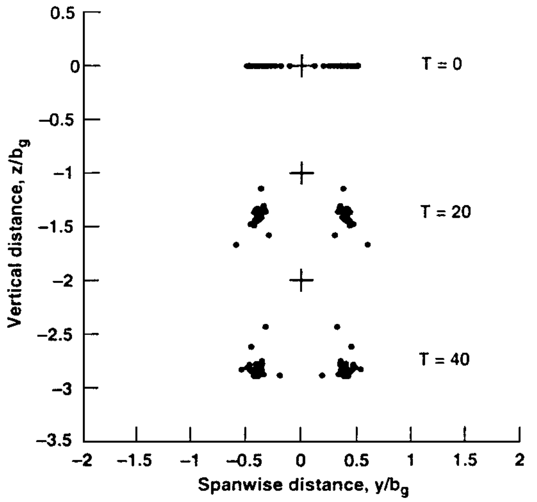

Fig. 10 Computation of rollup of vortex wake shed by single aircraft wing, $T = 0, 20$, and 40.

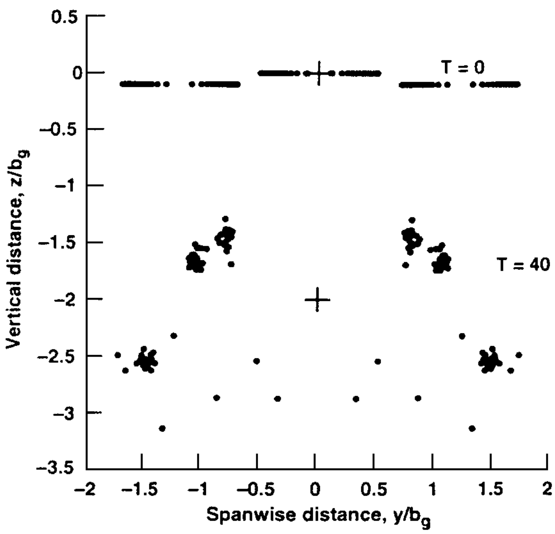

Fig. 11 Computation of rollup of vortex wakes shed by three aircraft wings flying side by side, $\Delta y_{\text{cntrln}} = 1.2b_g$, $\Delta z_{\text{cntrln}} = -0.1b_g$, and $T = 0$ and 40.

When an aircraft wing with the same span loading is placed off to both the port and starboard sides of the center wing, the rollup of the shed vortices again results in tight vortex cores (Fig. 11). Now, however, and as expected, the motion of the cores is different from that of the isolated wing, but the vortex core regions where circulation accumulates move about the same as that shown in Fig. 8b for three point vortices. Comparison of Fig. 11 and 8b tends to assure that the bulk of the circulation still forms a small concentrated vortex core and that the cores formed do not move appreciably differently from those determined by computations using point vortices. Because a number of additional approximations were also made in the foregoing analysis, experimental study and fine tuning of aircraft spacings with flow visualization will probably be required to verify and fine tune the most effective spacings for actual flight formations.

Multiple Formations

So far, the analysis has considered only one row or chevron of aircraft in the formations considered. From a wake–vortex safety point of view, the preliminary study conducted here indicates that vortex encounters will not occur if additional chevrons or rows of aircraft are added to a single row, if spacing guidelines are followed. For example, because the first or leading formation is designed so that inboard vortices convect upward rapidly, the region above it will again not be safe to enter until the vortex wakes have decomposed to a harmless level. The region below the leading formations, however, should be free of vortices everywhere except near the locations of the two outboard vortices, which are moving downward slowly. Therefore, prevention of vortex encounters requires that following chevrons each reduce the spanwise extent of the second and following formations so that none comes near the regions occupied by the outboard vortices of preceding formations (Fig. 12). As shown in Fig. 12, each succeeding formation has its spanwise extent reduced by two aircraft, so that if the first formation is composed of eight aircraft, following formations would each be composed successively of six, four, and two aircraft for a total of 20 aircraft (Fig. 12). Note that the spacings shown in Fig. 12 are not to scale. Of course, it is necessary to carry out a design analysis on every configuration to find its optimum wake-avoidance configuration before attempts are made to implement the design.

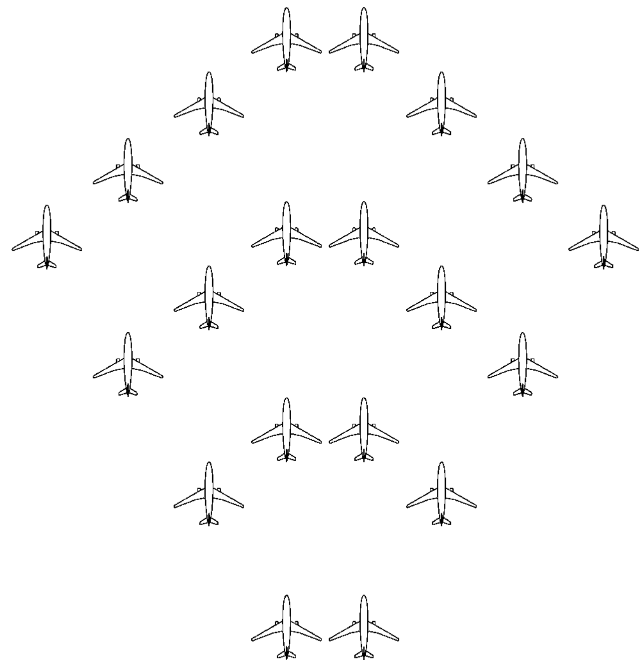

Fig. 12 Plan view of flight formations in tandem arranged so that neither the aircraft nor the parachutes will encounter a vortex nor parachutes deployed by preceding aircraft.

Because some airports are now operating near their capacity to accommodate landings and takeoffs, the possible use of formations of aircraft to expedite landings or takeoffs should be considered. The availability of wide runways is, of course, required along with careful control of vortex paths (G. W. Condon, "Large Concrete Surfaces to Permit Flexibility in Landing and Takeoff Operations for Increased Airport Capacity," private communication, July 2000). Although formations such as those studied here may provide a vortex-free region for closely following aircraft, a number of other considerations needs to be included in a more complex analysis to determine whether such a process is feasible or not.

Lifetime of Wake Generated by Formations

The inviscid analysis presented here redirects the lift-generated vortices shed by a flight formation, but does not indicate how the vortices decay nor how long the decay process will take. All of the upwardly induced vortex motions occur as closely spaced vortex pairs (Fig. 8). The question is how far will the vortex pairs rise and how long will they persist in an actual flight situation. The answer to that question is suggested by a number of experiments that led to the presence of closely spaced vortices. It was found that not only do the closely spaced pairs move rapidly, but they also quickly undergo several instabilities. A short-wave instability identified and analyzed by Leweke and Williamson[16] appears to begin shortly after

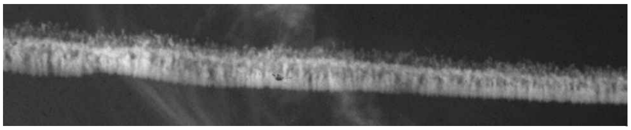

Fig. 13 Photograph of side view of condensation wake of subsonic transport at cruise altitude to illustrate circumferential striations brought about by short-wavelength elliptic instability in a vortex pair.[14]

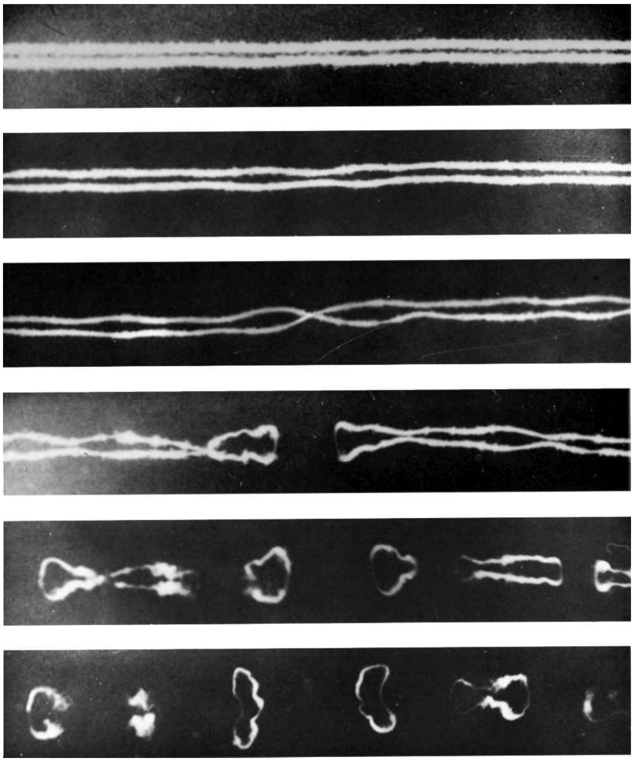

Fig. 14 Photographs taken at 15-s intervals from below condensation wake of B-47 in cruise configuration to illustrate mutually induced long-wavelength instability of a vortex pair (from Crow[15]).

the vortex pair has been generated. The visual characteristic that identifies the elliptic instability is the closely spaced circumferential striations around the outside of the condensation cloud that encloses the vortex pair (Fig. 13). The short-wavelength or elliptic instability appears to diffuse the outer parts of the flowfield surrounding the vortex pair, but may fail to make the organized intense inner parts of the vortex pair incoherent. In fact, observations of condensation wakes at cruise altitudes[14] indicate that the short-wave instability may prepare the inner part of the wake for growth of the long-wave instability.

As the name implies, the long-wave instability is characterized by lateral and vertical displacement waves about 6–8 wingspans along the length of the vortex filaments.[15] The waves are initiated by small disturbances in the flowfield of the wake and then grow quickly with time due to the mutually induced velocity field of the sinuous vortex filaments. As the waves increase in amplitude to one or more spans, they soon become large enough for the filaments to join across the span to form irregularly shaped vortex loops (Fig. 14). The irregularly shaped loops of vortex lines and their subsequent self-induced motions are very effective in accelerating the decomposition of the vortex wake so that its flowfield is incoherent and nonhazardous for in-trail penetrations. The long-wave instability was correctly explained and analyzed by Crow[15] and is the most destructive self-induced mechanism that occurs in a vortex pair.

In summary, experimental observations of vortex instabilities indicate that the motion of inboard vortex pairs shown in Fig. 8 will degenerate into flowfields that become incoherent quite quickly (less than a minute) after the vortices have risen a short distance above the flight level of the formation. The resulting incoherent regions will be highly turbulent and should disintegrate and decay rapidly to blend eventually with the surrounding atmosphere. Such a process is probably the one that will occur, but flight experiments will need to be conducted to determine the timescale and final disposition of the assembly of vortex pairs.

A comparable estimate for the decomposition and decay of the two outboard vortices is not presented in the literature. Because the two outboard vortices are descending slowly, the turbulence from the decomposition of the inboard pairs may spread enough to enhance the decomposition of the two most outboard vortices. However, the time required for the decay of the entire vortex systems being shed by the aircraft formations treated here is uncertain and needs to be explored experimentally. At the very worst, measurements taken of vortices at airports indicate that vortices usually decay to a harmless incoherent state in 2–3 min after being generated.

Conclusions

The results presented indicate that it is aerodynamically feasible to design formations of aircraft so that only the two most outboard vortices descend and all of the inboard vortices shed by an aircraft formation move rapidly upward. Because the two outboard vortices move downward slowly, the airspace below the formation has a large region that is free of vortices, so that parachutes can safely be deployed and descend to the ground. The numerical solutions presented here indicate that the vortex-redirection concept can be made to work if the distances between the centerlines of adjacent aircraft is kept between about $1.1b_g$ and $1.4b_g$ in the lateral and between $-0.025b_g$ and $-0.15b_g$ in the vertical. It is recommended that midrange values of horizontal and vertical spacings between the centerlines of adjacent aircraft in a flight experiment begin with values of $1.2b_g$ and $-0.1b_g$, respectively. Although the range of permissible values are several tenths of a wingspan in both lateral and vertical spacings, it is recommended that all aircraft spacings in a given formation be restrained to the same values so that intentional and inadvertent spacings do not vary from the ideal by more than $\pm 0.01b_g$. Although these lateral and vertical separation distances between adjacent aircraft are small, they are large enough that electronic station-keeping equipment should be able to maintain the required spacings. Variables such as utilization of aircraft of different sizes in a formation and atmospheric turbulence were not included in the present study. Some of the parameters that were ignored may not have a significant affect on vortex redirection, but others will need to be addressed before the vortex redirection concept can be implemented onto flight hardware.

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